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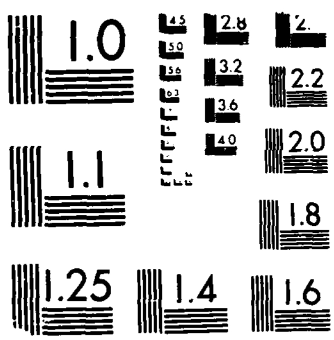

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Annual Report

Development of Adaptive Grid Schemes based on Poisson Grid Generators

AFOSR - 85 - 0195

Principal Investigator
Dale A. Anderson

Major Staff
Stephen Kennon

Research Assistant
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The University
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Arlington, Texas

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION / AVAILABILITY OF REPORT APPROVED FOR PUBLIC RELEASE DISTRIBUTION IS UNLIMITED
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 88-0126
6a. NAME OF PERFORMING ORGANIZATION UNIV OF TEXAS AT ARLINGTON 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION AFOSR/NA
6c. ADDRESS (City, State, and ZIP Code) ARLINGTON TEXAS 76019	7b. ADDRESS (City, State, and ZIP Code) BUILDING 410 BOLLING AFB, DC 20332-6448
8a. NAME OF FUNDING / SPONSORING ORGANIZATION AFOSR 8b. OFFICE SYMBOL (If applicable) NA	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-85-0195
8c. ADDRESS (City, State, and ZIP Code) BUILDING 410 BOLLING AFB, DC 20332-6448	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 61102F PROJECT NO. 2307 TASK NO. A1 WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
(U) DEVELOPMENT OF ADAPTIVE GRID SCHEMES BASED ON POISSON GRID GENERATORS

12. PERSONAL AUTHOR(S)
DALE A. ANDERSON

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
ANNUAL REPORT	FROM 11/15/86 TO 11/14/87	87/12/10	19

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	ADAPTIVE GRIDS, GRID GENERATORS

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Research activities centered on construction of adaptive grids using poisson grid generators are summarized in this report. Major areas include adaptive area/volume control, orthogonal adaptive grids in two dimensions, adaptive unstructured grids and linear methods for adaptive grids.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL JAMES M MCMICHAEL	22b. TELEPHONE (Include Area Code) 202-767-4936 22c. OFFICE SYMBOL AFOSR/NA

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83 APR edition may be used until exhausted.
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Abstract

A summary of the technical work performed during the past twelve months under AFOSR Grant 85-0195 is presented in this report. Significant progress on a number of adaptive concepts has been made. Problems associated with earlier adaptive mesh schemes controlling cell area/volume have been resolved. Resolution of this difficulty led to the evolution of an adaptive orthogonal scheme for two-dimensional grids. This technique is even structurally simpler than the original Poisson equation based adaptive methods. Other research emphasis has been placed on developing an unstructured solution scheme as a fast solver to generate these grids.

Introduction

The work schedule for the first year as presented in the original proposal is shown below for completeness.

Proposed first year activities:

1.1 One-dimensional grid speed calculations will be completed. Best method for general time-dependent calculations will be selected.

1.2 Best technique for advancing grid in time accurate calculations will be selected from the one-dimensional experiments.

1.3 First attempts will be made at construction of a general two-dimensional adaptive routine for multiple block systems.

1.4 Collaboration will be initiated with AFWAL on incorporating adaptive technology in existing codes.

1.5 An analysis of appropriate error measures for use in adaptive grid studies will be made.

Actual research topics deviated from those originally considered due to unresolved issues remaining from earlier thoughts and new ideas and potential applications which were judged to be more important. A brief review of each of four separate areas of research explored over the past year is presented in the next section.

Research Status

1. Adaptive Grid Scheme for Controlling Cell Area/Volume

In January, 1987, a paper was presented at Reno AIAA meeting (Ref 1) relating the cell area or volume to the grid control functions of the Thompson scheme. By way of review, the Poisson grid generator proposed by Alan Winslow in 1981 introduced a diffusion, D, through the grid laws

$$\nabla \cdot (D\nabla\xi) = 0$$

$$\nabla \cdot (D\nabla\eta) = 0$$

The diffusion controls the mapping relating the computational coordinates.

If I is defined as

$$I = \xi_x \eta_y - \xi_y \eta_x$$

The Laplacian of I may be formed to relate the differential cell area ($1/I = J$) to the diffusion term. Thus,

$$\nabla^2 - \quad \xi_x \frac{\partial \nabla^2 \eta}{\partial y} + \eta_y \frac{\partial \nabla^2 \xi}{\partial x} - \eta_x \frac{\partial \nabla^2 \xi}{\partial \eta} - \xi_y \frac{\partial \nabla^2 \eta}{\partial x}$$

$$+ 2 \left[\frac{\partial \nabla \xi}{\partial x} \cdot \frac{\partial \nabla \eta}{\partial y} - \frac{\partial \nabla \eta}{\partial x} \cdot \frac{\partial \nabla \xi}{\partial y}\right]$$

If the grid is not highly skewed, the last bracketed expression may be assumed to be small and a differential equation of the form

$$\nabla^2(\bar{J} - \bar{D}) - \nabla \bar{J} \cdot \nabla (\bar{J} - \bar{D}) = 0$$

where barred quantities represent ln and $J = 1/I$.

A solution of this equation is

$$\bar{J} = \text{constant} \times \bar{D}.$$

In the original paper, the cross term in the brackets of the expansion of $\nabla^2 I$ was omitted. This error necessitated a search for the reason that known analytic mappings did not satisfy the Jacobian diffusion relation. When the cross term is included, numerical and analytical results agree. However, the use of the $J = KD$ approximation as an adaptive measure works very well and is a viable way of providing an adaptive grid.

2. Othogonal, Adaptive Grids

An adaptive grid that is also orthogonal provides a number of computational advantages. In addition to simplification of boundary condition application, the elimination of off-diagonal terms in numerical algorithms is an important advantage. Conditions for generating orthogonal grids have been well known for a number of years but the ability to produce adaption was only added during the past few months of the present grant period.

If the Thompson scheme is written assuming that the resulting grid in physical space is orthogonal, the cross-derivative term is 0. This leads to an expression of the form

$$\propto (\vec{r}_{\xi\xi} + \vec{r}_{\xi}\phi) + \gamma\,(\vec{r}_{\eta\eta} + \vec{r}_{\eta}\psi) = 0$$

where

$$\vec{r} = (x,y)^T$$

$$\propto = x_\eta^2 + y_\eta^2 = S_\eta^2$$

$$\gamma = x_\xi^2 + x_\eta^2 = S_\xi^2$$

and ϕ and ψ are the Thomas and Middlecoff grid control functions.

Let

$$f^2 = \frac{\propto}{\gamma} = \frac{S_\eta^2}{S_\xi^2}$$

Then

$$\vec{r}_{\xi\xi} + \vec{r}_{\xi}\phi + \frac{1}{f^2}\,[\vec{r}_{\eta\eta} + \vec{r}_{\eta}\psi] = 0$$

However, the theory of quasi-conformal mapping assures that an orthogonal map is produced by solving the equation

$$\partial/\partial\xi\ (f\vec{r}_{\xi}) + \frac{\partial}{\partial\eta}\,(\frac{1}{f}\,\vec{r}_{\eta}) = 0$$

subject to appropriate boundary conditions. In this expression, f is the conformal module or the cell aspect ratio as previously noted. If this generating equation is expanded

$$\vec{r}_{\xi\xi} + \frac{f_\xi}{f}\,\vec{r}_{\xi} + \frac{1}{f^2}\,[\vec{r}_{\eta\eta} - \frac{f_\eta}{f}\,\vec{r}_{\eta}] = 0$$

Thus, the conformal module is related to the usual ϕ and ψ (P,Q) of the Poisson schemes by the expressions

$$\phi = \frac{f_\xi}{f} \quad , \quad \psi = -\frac{f_\eta}{f}$$

The key point is to note that f is the ratio of arc lengths. Consequently, these lengths may be adjusted by using some weight function scheme. Thus, if

$$f = S_\eta / S_\xi$$

and an arc equidistribution scheme is employed to evaluate the variation in arc lengths,

$$S_\xi W_1 = \text{Constant} = C_1$$

$$S_\eta W_2 = \text{Constant} = C_2$$

then

$$f = \frac{W_1}{W_2}\ \frac{C_2}{C_1}$$

The constants are easily obtained by noting that f = 1 in regions with no adaption.

This method has been applied to several simple geometric problems and works surprisingly well. Continued development of this technique is necessary.

While the two-dimensional adaptive orthogonal scheme works well, the same ideas in three dimensions cannot be employed. When orthogonality is required in three dimensions, the off-diagonal terms in the metric tensor are set to zero. At this point, all options on control of the grid have disappeared and no adaption is possible. If however, the restriction to Euclidian space is removed, the new concepts may be applied. The problem is that mappings in non-Euclidian space not only influence geometry but typical solvers are altered. As a result, any venture into this area necessitates more than a simple grid mapping study.

3. Linear Elliptic Adaptive Grid Generation

Several popular grid generation methods (Anderson(1) diffusion, Thompson(2), Winslow(3)) are based on a set of linear elliptic governing equations. However, these linear equations are transformed from physical (x,y)-space to computational (ξ,η)-space. The transformed equations then become non-linear, and are solved in computational space for the (x,y) coordinates of the grid points. Although the original equations are linear and uncoupled, the transformed equations are non-linear and strongly coupled. Therefore, the possibility of solving the original linear equations in physical space was examined, resulting in the development of a new adaptive grid generation method.

The basis for this new adaptive grid generation method is solving a set of linear elliptic partial differential equations in physical space. As such, it is necessary to have an initial, base grid on which the solution is to be found. This base grid can be generated by any method as long as it is reasonably smooth and non-overlapped. Once the base grid is defined, one can generate grids that are smooth, adapted to specified weight functions, and orthogonal at specified boundaries. The basic elliptic equation set is that of the diffusion grid generation method.

The diffusion method of Anderson(1) has proven to be a very powerful adaptive grid generation method. The governing equations are given by the set

$$\begin{aligned} \nabla \cdot D\nabla\xi &= 0 \\ \nabla \cdot D\nabla\eta &= 0 \end{aligned} \tag{1}$$

where D(x,y) is the 'diffusion' function. The diffusion function D(x,y) can be shown to be directly related to the local cell area (i.e. the Jacobian) of the grid that results from solving the diffusion equation set. Therefore, if one wishes to have a grid that adapts to a positive weight function $W(x,y)$, then one should chose the diffusion function such that

$$DW = \text{constant} \tag{2}$$

Thus, where the weight function is large, the diffusion will be small, and in turn the local grid spacing will be small. There are several advantages to this method:

1. The method satisfies an extremum principle for any choice of $D(x,y) \geq 0$. This means that the method is guaranteed to produce grids without overlap.
2. Only one grid control function (namely D) need be chosen. This can be contrasted to other elliptic-based methods that require the specification of two grid control functions (e.g. the Thompson scheme requires the choice of two functions P and Q).
3. The diffusion equations can be transformed into equations that are identical to the Thompson equations where:

$$\nabla^2\xi = P(x,y) = \frac{-\nabla D}{D} \cdot \nabla\xi$$

$$\nabla^2\eta = Q(x,y) = \frac{-\nabla D}{D} \cdot \nabla\eta$$

The normal practice when solving elliptic grid generation equations is to interchange the dependent (ξ,η) and independent (x,y) variables to get an equation set of the form (for the diffusion method):

$$\alpha x_{\xi\xi} - 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = (\alpha x_{\xi} - \beta x_{\eta})\frac{D_{\xi}}{D} - (\beta x_{\xi} - \gamma x_{\eta})\frac{D_{\eta}}{D}$$

$$\alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = (\alpha y_{\xi} - \beta y_{\eta})\frac{D_{\xi}}{D} - (\beta y_{\xi} - \gamma y_{\eta})\frac{D_{\eta}}{D}$$

where

$$a = x_\eta^2 + y_\eta^2$$

$$B = x_\xi x_\eta + y_\xi y_\eta$$

$$\gamma = x_\xi^2 + y_\xi^2$$

One can see that these equations are non-linear and coupled due to the presence of the factors a, B, γ which depend on both x and y. Instead of solving this non-linear set of equations in computational space, it was found that one could solve the linear uncoupled set (eq. 1) in physical space. The advantages of this approach are:

1. Since the governing equation set is linear, elliptic and satisfies an extremum principle, there exists a unique solution.
2. Standard iterative methods (such as successive overrelaxation, conjugate gradient, and others) will be guaranteed to converge regardless of the initial guess for the solution.
3. The possibility exists for using very fast direct solution techniques such as cyclic reduction and fast-fourier-transform type schemes. Therefore, the adaptive computational grid could be computed in one direct-solution step, without iteration.
4. Implementation of Neumann-type boundary conditions becomes almost trivial. In many cases, one would like the grid lines to be orthogonal at certain boundaries of the domain. This results in a Neumann-type boundary condition which can be imposed directly on the linear grid equations. This is opposed to methods that solve the equations in computational space. In that case, the grid points must slide along the boundary and therefore the boundary must be spline-fit after each iteration. This incurs additional expense and program complexity.

5. No interpolation is needed during each iteration to update the values of the weight function at the grid points. In a standard method, the weight function must be updated after each iteration, which is a very costly procedure.

The real advantage of this method will be experienced in the generation of three-dimensional adaptive grids. Standard three-dimensional adaptive grid generation methods require that the grid points slide along boundaries. This requires a sophisticated surface definition method (such as bi-cubic patches). On the other hand, the new linear adaptive method does not require the movement of points along any boundary.

The method was tested on a set of model problems set in the unit square. The first example (fig. 1) shows the result of adaption to the function

$$W(x,y) = \begin{matrix} 1 & \text{if } r < \frac{1}{4} \\ 2 & \text{if } r \geq \frac{1}{4} \end{matrix} \qquad (3)$$

where

$$r = (x - \frac{1}{2})^2 + (y - \frac{1}{2})^2$$

The grid adapts well to the weighting function and is clearly uniform where W is uniform. The second example (fig. 2) involves adaption to a model of a shock wave where the weighting function is given by $W = 1 + 10|\nabla u|^2$ where

$$u = \tanh(2x - y) \qquad (4)$$

The grid adapts to the model shock wave in a reasonably smooth manner. It should be noted that the grid was specified to be orthogonal to all boundaries.

The linear adaptive grid generation method has also been tested on realistic turbomachinery problems. In all cases it has proven to be robust and efficient and should find wide application in computational fluid dynamics.

4. Generation of Unstructured Triangular Grids Using Elliptic Partial Differential Equations

The past few years have seen a growing interest in the use of unstructured triangular grids in computational fluid dynamics (cf. Jameson [4], Lohner [5]). One of the major difficulties in the generation of unstructured grids is to produce grids that are smooth, non-overlapped and adaptive to a specified weight function. Therefore, a new method has been developed for generating adaptive unstructured triangular grids based on elliptic partial differential equations (PDE). The use of an elliptic PDE ensures that the resulting grid is non-overlapped and smooth, while adaption is achieved by using Anderson's diffusion method [3].

The new adaptive grid generation method is based on solving the equations (see above discussion for more details on the diffusion method)

$$\nabla \cdot D\nabla\xi = 0$$

$$\nabla \cdot D\nabla\eta = 0 \qquad (1)$$

These equations are transformed to computational space to give a set of equations to be solved for the (x,y)-coordinates of the grid:

$$a x_{\xi\xi} - 2B x_{\xi\eta} + \gamma x_{\eta\eta} = (a x_\xi - B x_\eta)\frac{D_\xi}{D} - (B x_\xi - \gamma x_\eta)\frac{D_\eta}{D}$$

$$a y_{\xi\xi} - 2B y_{\xi\eta} + \gamma y_{\eta\eta} = (a y_\xi - B y_\eta)\frac{D_\xi}{D} - (B y_\xi - \gamma y_\eta)\frac{D_\eta}{D}$$

where

$$a = x_\eta^2 + y_\eta^2$$

$$B = x_\xi x_\eta + y_\xi y_\eta$$

$$\gamma = x_\xi^2 + y_\xi^2$$

For an unstructured grid, the computational space would also be unstructured (as opposed to a structured grid that produces a uniform grid in computational space). Therefore, instead of transforming to computational space, we use the concept of a master element [6]. Each triangular element in physical space is mapped to the single triangular master element. All computations are done on the master element and then mapped back to physical space. The governing equations are discretized using a standard finite element procedure and solved by simple point relaxation. The method was tested on a model problem set in the unit circle. The initial unstructured grid is shown in fig. 3 and was generated using a Delaunay triangulation procedure [7]. Figure 4 shows the result of requiring that the grid adapt to the weight function

$$W(x,y) = \begin{matrix} 10 & \text{if } r < \frac{1}{4} \\ \frac{1}{10} & \text{if } r \geq \frac{1}{4} \end{matrix} \qquad (2)$$

where

$$r = (x - \frac{1}{2})^2 + (y - \frac{1}{2})^2$$

Note that where the weight function is large, the grid cells are small, and vice versa. Also note that the grid is relatively uniform where the weight function is uniform

This research has shown that methods that have been successfully used to generate structured adaptive grids can be applied to generate unstructured adaptive grids. The new unstructured grid adaption method should find wide applicability in the new flow solvers being developed for triangular grids.

References:

1. Anderson, D.A., "Adaptive Grid Scheme Controlling Cell Area/ Volume," AIAA Paper No. 87-0202, Reno, Nevada, January, 1987.

2. Thompson, J.F., F.C. Thames, and C.M. Mastin, "Automatic Numerical Generation of Body Fitted Curvilinear Coordinate Systems for Fields Containing any Number of Arbitrary Two-Dimensional Bodies," Journal of Computational Physics, Vol. 15, 1974, pp. 299-319.

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6. Becker, E.B., G.F. Carey, and J.T. Oden, "Finite Elements: An Introduction," Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1981.

7. Kennon, S.R., "Numerical Solution of Weak Forms of Conservation Laws on Optimal Unstructured Triangular Grids," Ph.D. Thesis, Department of Aerospace Engineering and Engineering Mechanics, University of Texas at Austin, 1987.

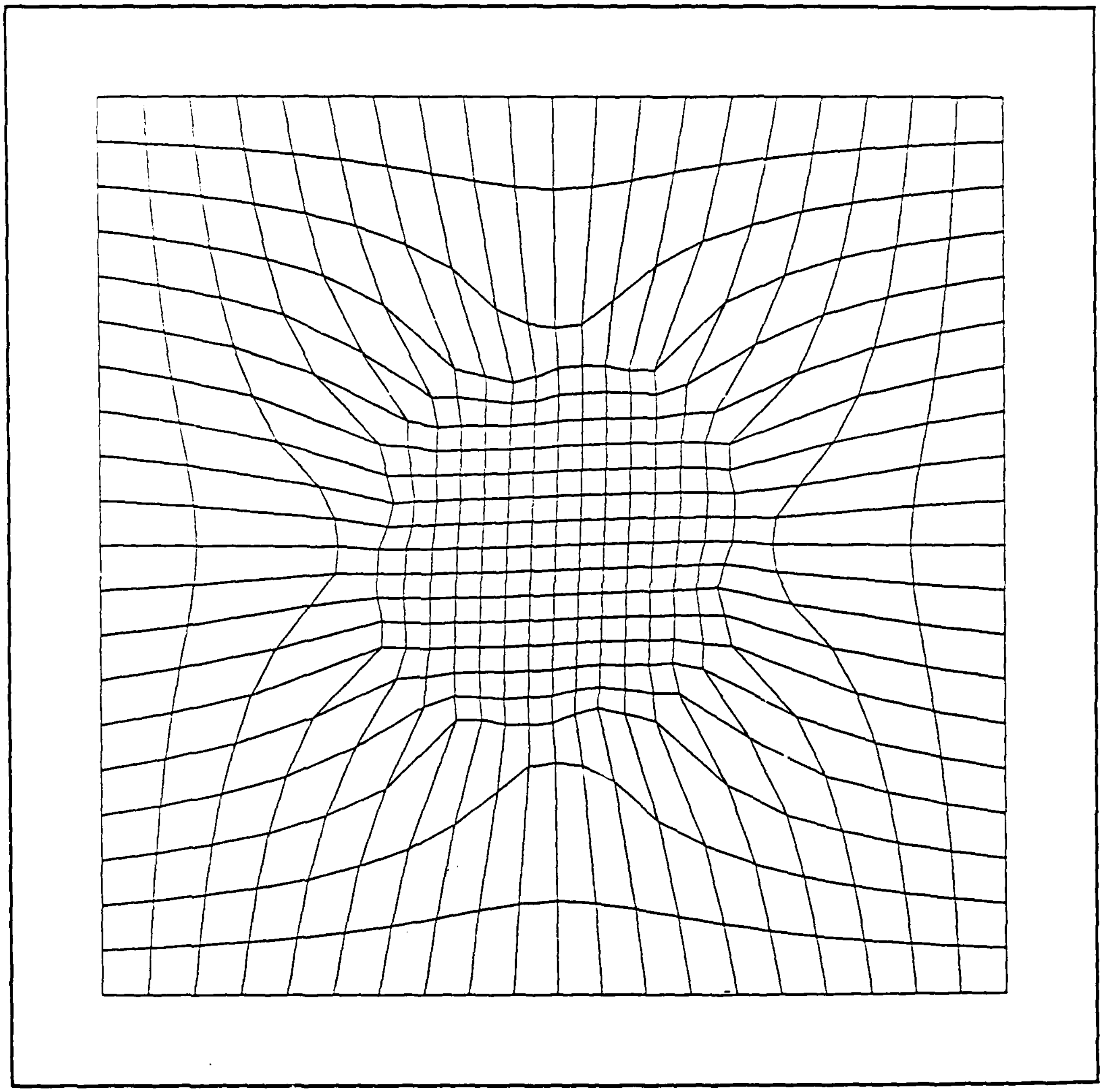

Figure 1 Linear adaptive grid generation-Example 1

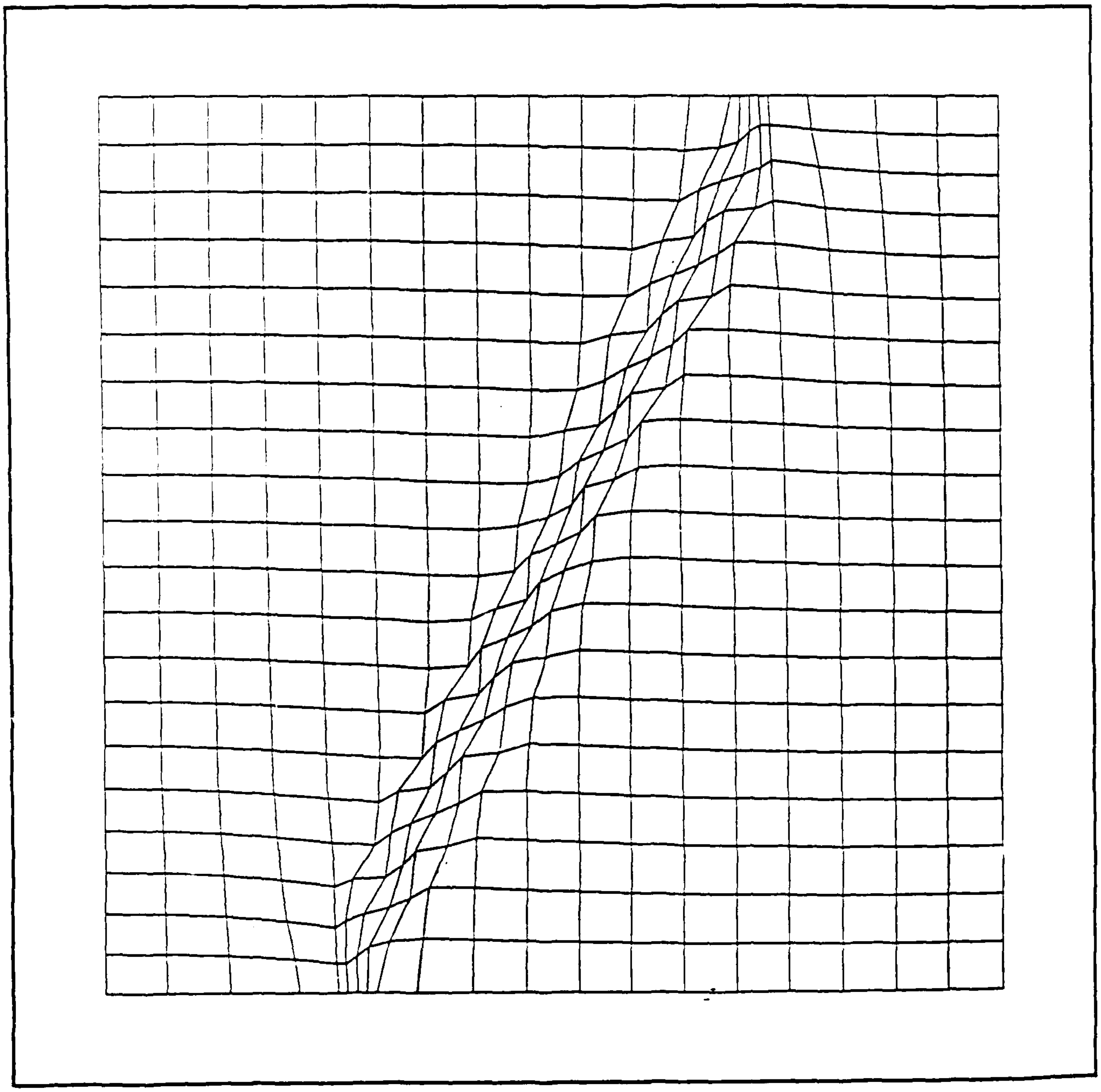

Figure 2 Linear adaptive grid generation-Example 2

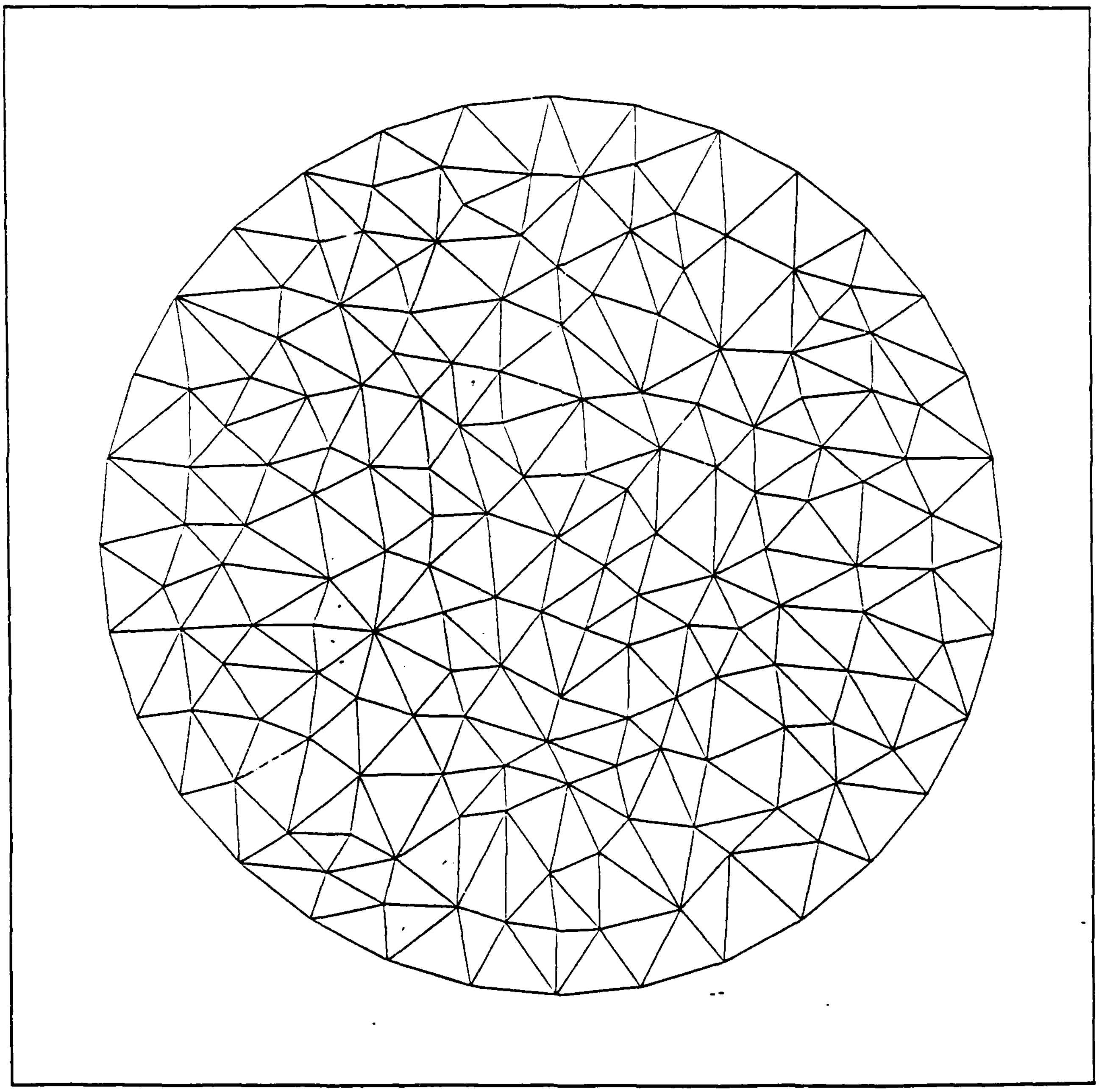

Figure 3 Unstructured Adaptive Grid Generation: Initial Grid

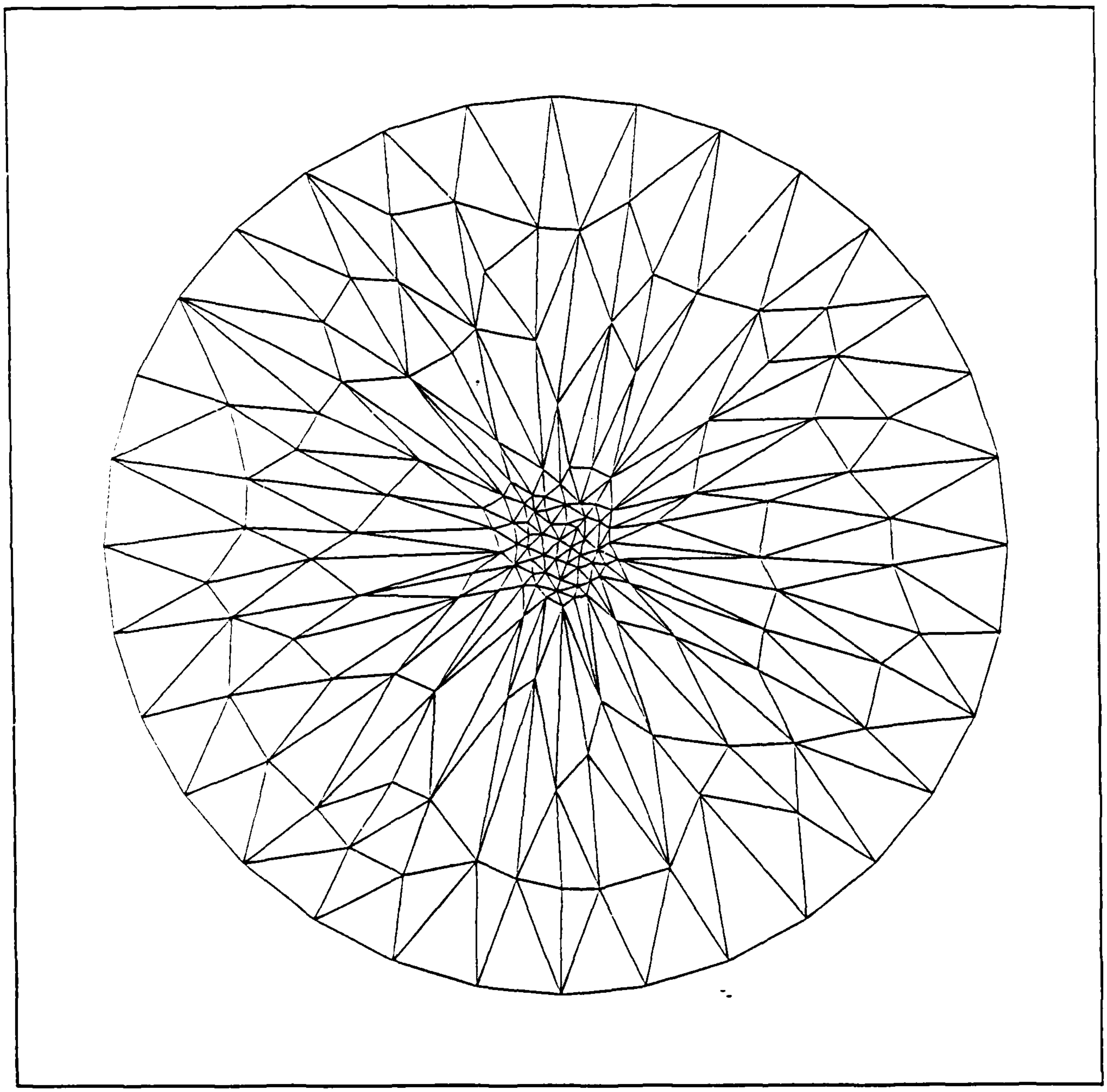

Figure 4 Unstructured Adaptive Grid Generation: Adapted Grid

Expected Progress During the Next Six Months

1. The diffusion formulation for adaptive grids will be submitted for publication in a journal as soon as a few more geometric examples of the application of the method are complete.
2. The work on orthogonal adaptive grids will be submitted for publication during the next six months. This work is important because it ties the solution of the Poisson grid generators directly to construction of two-dimensional orthogonal adaptive grids with a minimum of change.
3. A paper on the linear adaptive grid solver will be submitted to a suitable conference. This approach shows promise in accelerating the grid generation procedure when Poisson grid generators are used.
4. Since unstructured grids appear to hold great promise, work will continue on development of unstructured solvers. In a previous report, work on finite-volume solvers was presented and finite-element solvers were applied here. Some decision must be reached on the best direction for continued research in this area.
5. In addition to the grid generation schemes discussed in this report, application to practical problems must receive attention. Work continues on applications to airfoils and it is expected that a number of results will be forthcoming for airfoils, wings, reentry configurations, and turbine and cascade flows.

6. The adaptive grid technology developed under this AFOSR sponsored program will be implemented in a new blocked grid generator. This interactive blocked grid generator is being developed by General Dynamics, Fort Worth under Air Force contract.

In addition to the expected publications noted in this section, the principal investigator has been invited to present a lecture on Application of Poisson Grid Generators to Problems in Fluid Dynamics at the 7th International Conference on Finite Element Methods in Flow Problems in 1989.

Publications either accepted or appearing over the past twelve months.

Anderson, Dale A., "An Adaptive Grid Scheme Controlling Cell Area/ Volume," presented at the 25th Aerospace Sciences meeting, Reno, Nevada. AIAA paper 87-0102, January, 1987.

Anderson, Dale A., "Equidistribution Schemes, Poisson Generators and Adaptive Grids," Applied Mathematics and Computation, December, 1987.

Noack, R.W. and Dale A. Anderson, "Solution Adaptive Grid Generation using Parabolic Partial Differential Equations," will be presented at the 26th Aerospace Sciences meeting, Reno, Nevada. AIAA paper 88-0315, January, 1988.

Professional Personnel on this Program

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